

**B.Sc. (H) Mathematics**  
**SEMESTER SCHEME ( 2011- ONWARDS)**  
(Academic Council Resolution 3 (17) dated 25.04.2011)

**DEPARTMENT OF MATHEMATICS**  
**UNIVERSITY OF DELHI**  
**DELHI-110007**

Following will be the contents of proposed syllabus of B.Sc. (Hons.) Mathematics in the session 2011-12 and onwards.

There are 19 papers of Mathematics (Algebra 5, Analysis 5, Calculus 2, Differential Equations & Mathematical Modeling 3, Numerical Methods & Programming 1, Probability and Statistics 1, Linear Prog. & Theory of Games 1, Optional Paper 1).

The Mathematics Honors degree will consist of 2600 Marks. Each theory paper will be of 100 marks. All the theory and practical papers will have examination of 3 hours duration. Paper containing practical components will be of 150 marks. The practical paper is common for all the papers in a particular semester. There will be an external examiner in all the practical exams. There will be 5 lectures and one tutorial for all the papers. Two classes are allotted for the practical per student per week. A practical group will consists of at most 20 students. Use of Scientific Calculator is allowed.

Every college is advised to offer at least two optional courses in Mathematics out of four courses (Discrete Mathematics, Mathematical Finance, Mechanics and Number Theory) in Semester VI.

***The semester-wise distribution of the papers is as follows:***

<b>SEMESTER I</b>	Calculus I	Analysis I	Algebra I	Qualifying paper
<b>SEMESTER II</b>	Diff. Eqn. I	Analysis II	Prob. and Stats.	Credit course I
<b>SEMESTER III</b>	Calculus II	Num. Methods	Algebra II	Credit course II
<b>SEMESTER IV</b>	Diff. Eqn. II	Analysis III	Algebra III	Credit course III.1
<b>SEMESTER V</b>	Diff. Eqn. III	Analysis IV	Algebra IV	Linear Prog. & Theory of Games
<b>SEMESTER VI</b>	Analysis V	Algebra V	Optional	Credit course III.2

Along with the above mentioned papers, a student will have to opt for four credit courses from disciplines other than Mathematics and a qualifying paper. A student will have to choose one course each from Credit Course I in semester I , Credit Course II in semester II and Qualifying Course in Semester III .A student will have to opt for two courses from Credit Course III as Credit III.1 and Credit Course III.2 in semester IV and Semester VI respectively. In those subjects where more than one course is offered, the student shall opt for one of the course. **But if a student opts for Physics-II he/ she may opt for Physics(Lab).** The marks of Credit Course I, II, III.1 and III.2 shall count in the final result of the student.

### Credit Course I

- (i) Ethics in Public Domain
- (ii) Environmental Issues in India
- (iii) Reading Gandhi
- (iv) The Individual and Society
- (v) Hindi Language, Literature and Culture
- (vi) Gender and Society
- (vii) Financial Management
- (viii) Chemistry
- (ix) Physics-I

Note: (a) Courses (i)-(vi) are the interdisciplinary courses of the BA (Hons)

Programme.

(b) Course (vii) is the elective course EL 210 (vi) of B.Sc Programme.

(c) Course (viii) is the Paper V being taught in First year Physics (Hons).

(d) Course (ix) is the Paper V being taught in First year Chemistry (Hons).

### Credit Course II

- (i) English
- (ii) Hindi
- (iii) Sanskrit
- (iv) Chemistry
- (v) Physics-I
- (vi) Chemistry (Lab)
- (vii) Physics (Lab)

Note: (a) Courses (i)-(iii) are the language credit courses of the BA (Hons)

Programme.

(b) Course (iv) is the Paper V being taught in First year Physics (Hons). For students of Maths (Hons), only six out of the twelve experiments will have to be done. These experiments may be selected at the college level.

(c) Course (v) is the Paper V being taught in First year Chemistry (Hons).

(d) Course (vi) is the Paper VIII being taught in First year Physics (Hons).

(e) Course (vii) is the Lab II being conducted in First year Chemistry (Hons).

#### Qualifying Course

- (i) English (Higher)
- (ii) English (Lower)
- (iii) Hindi (Higher)
- (iv) Hindi (Lower)
- (v) Sanskrit
- (vi) Chemistry (Lab)
- (vii) Physics (Lab)

Note: (a) Courses (i)-(v) are the language qualifying courses of the BA (Hons)

Programme.

(b) Course (vi) is the Paper VIII being taught in First year Physics (Hons).

(c) Course (vii) is the Lab II being conducted in First year Chemistry (Hons).

#### **MARKS:**

\* Each of the Credit I, Credit II and Qualifying courses are of 100 marks:

Semester examination 75 marks, internal assessment 25 marks.

\* The pass mark for the credit courses is 40 percent.

\* The pass mark for the qualifying courses is 36 percent. A student has to pass in the

qualifying course to be eligible for an Honors degree. However, the marks in this course will not be counted in the final division awarded.

- \* Internal assessments will be held for the credit courses but not for the qualifying course.

#### **NUMBER OF LECTURES:**

- \* Four hours per week or four classes for Credit Course I , Credit Course II and Qualifying Course (i)-(v). For each of **the credit** courses, one tutorial will be held for students. Six hours for ( vi) and ( vii).

#### **RULES:**

- \* Every student must opt for at least one language. It can either be a credit course or a qualifying course. If they are opting for a language in both the credit as well as the qualifying course then these cannot be the same languages.
- \* A student offering Chemistry/ Physics-I as Credit Course I will not be allowed to offer the same as Credit Course II.
- \* **A student offering Chemistry/ Physics-I as Credit Course I can opt for Chemistry (Lab)/ Physics (Lab) as Credit Course II but then they cannot opt for these courses as qualifying courses.**
- \* A student will be allowed to take Chemistry (Lab)/ Physics (Lab) as a qualifying course if they have opted for Chemistry/Physics-I respectively as a credit course.

#### Credit Course III

- (i) Psychology for Living
  - (ii) Hindi Literature
  - (iii) Modern Indian Literature, Poems and Short Stories; Novel or Play
- OR

Cultural Diversity, Linguistic Plurality and Literary Traditions in India.

- (iv) Formal Logic/ Symbolic Logic
- OR

Readings in Western Philosophy

OR

Theory of Consciousness

- (v) Citizenship in Globalizing World
  - (vi) Culture in India: a Historical Perspective
- OR

Delhi: Ancient, Medieval and Modern

OR

Religion and Religiosity in India

OR

Inequality or Difference in India

- (vii) Sociology of Contemporary India
  - (viii) Principles of Geography
- OR

Geography of India

- (ix) Principles of Economics
- (x) Financial Accounting
- (xi) Green Chemistry
- (xii) Biotechnology
- (xiii) Physics- II
- (xiv) Biophysics
- (xv) Physics (Lab)

Note: (a) Courses (i)-(ix) are the discipline centred courses of the BA (Hons)

Programme.

- (b) Course (x-xii) are the elective courses EL 210 (v), EL 310 (i) and EL 310 (iii) of B.Sc Programme.
- (c) Course (xiii) is the Paper XI being taught in Second year Chemistry (Hons).
- (d) Course (xiv) is the Paper XXII (Option 2) being taught in Third year Physics (Hons).
- (e) Course (xv) is Lab V being conducted in Second year Chemistry (Hons)

**MARKS:**

- \* Each course carries 100 marks: examination 75 marks, internal assessments 25 marks.
- \* The pass mark is 40 percent.

**NUMBER OF LECTURES:**

- \* Four hours per week or Four classes for courses (i-xiv). For each of these courses, one tutorial will be held for students.
- \* Six hours or Six classes for course (xv).

## **SEMESTER I**

### **I.1 Calculus I**

Total marks: 150

Theory: 75

Practical: 50

Internal Assessment: 25

5 Lectures, 2 Practicals, 1 Tutorial (per week per student)

Hyperbolic functions, higher order derivatives, Leibniz rule and its applications to problems of type  $e^{ax+b}\sin x$ ,  $e^{ax+b}\cos x$ ,  $(ax+b)^n\sin x$ ,  $(ax+b)^n\cos x$ , concavity and inflection points, asymptotes, curve tracing in Cartesian coordinates, tracing in polar coordinates of standard curves, L'Hopital's rule, applications in business, economics and life sciences.

*References:*

[1]: Chapter 4 (Sections 4.3-4.5 (page 124-157), 4.7).

[2]: Chapter 7 (Section 7.8), Chapter 11 (Section 11.1).

Reduction formulae, derivations and illustrations of reduction formulae of the type  $\int \sin^n x dx$ ,  $\int \cos^n x dx$ ,  $\int \tan^n x dx$ ,  $\int \sec^n x dx$ ,  $\int (\log x)^n dx$ ,  $\int \sin^n x \cos^m x dx$ , volumes by slicing, disks and washers methods, volumes by cylindrical shells, parametric equations, parameterizing a curve, arc length, arc length of parametric curves, area of surface of revolution.

*References:*

[1]: Chapter 9 (Sections 9.4 (Pages 471-475 (excluding lines in  $R^3$ ))).

[2]: Chapter 8 (Sections 8.2-8.3 (pages 532-538 (excluding integrating products of tangents and secants))), Chapter 6 (Section 6.2-6.5 (excluding arc length by numerical methods))

Techniques of sketching conics, reflection properties of conics, rotation of axes and second degree equations, classification into conics using the discriminant, polar equations of conics.

*Reference:*

[2]: Chapter 11 (Sections 11.4-11.6 (up to page 775 excluding sketching conics in polar coordinates)).

Triple product, introduction to vector functions, operations with vector-valued functions, limits and continuity of vector functions, differentiation and integration of vector functions, tangent and normal components of acceleration, modeling ballistics and planetary motion, Kepler's second law.

*Reference:*

[1] Chapter 9 (Section 9.3 (pages 468-469)), Chapter 10

***Practical / Lab work to be performed on a computer:***

Modeling of the following problems using *Matlab / Mathematica / Maple* etc.



- (i) Plotting of graphs of function  $e^{ax+b}$ ,  $\log(ax+b)$ ,  $1/(ax+b)$ ,  $\sin(ax+b)$ ,  $\cos(ax+b)$ ,  $|ax+b|$  and be able to find the effect of  $a$  and  $b$  on the graph.
- (ii) Plotting the graphs of polynomial of degree 4 and 5, the derivative graph, the second derivative graph and comparing them.
- (iii) Any one of the following
- Sketching parametric curves (Eg. Trochoid, cycloid, epicycloids, hypocycloid)
  - Obtaining surface of revolution of curves
  - Tracing of conics in Cartesian coordinates/ polar coordinates
  - Sketching ellipsoid, hyperboloid of one and two sheets, elliptic cone, elliptic paraboloid, hyperbolic paraboloid using Cartesian co-ordinates.
- (iv) Any one of the following
- To find numbers between two real numbers.
  - Plotting subsets of  $R$  to study boundedness /unboundedness and bounds (if they exist).
  - Plotting of sets on  $R$  to discuss the idea of cluster points,  $\lim \sup$ ,  $\lim \inf$ .
- (v) Any one of the following
- Plotting of recursive sequences.
  - Study the convergence of sequences through plotting.
  - Verify Bolzano Weirstrass theorem through plotting of sequences and hence identify convergent subsequences from the plot.
  - Studying the convergence / divergence of infinite series by plotting their sequences of partial sum.
- (vi) Any one of the following
- Cauchy's root test by plotting  $n^{\text{th}}$  roots
  - Ratio test by plotting the ratio of  $n^{\text{th}}$  and  $n+1^{\text{th}}$  term.
- (vii) Matrix operation (addition, multiplication, inverse, transpose)

#### REFERENCES:

- M. J. Strauss, G. L. Bradley and K. J. Smith**, *Calculus* (3<sup>rd</sup> Edition), Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2007.
- H. Anton, I. Bivens and S. Davis**, *Calculus* (7<sup>th</sup> Edition), John Wiley and Sons (Asia) Pte. Ltd., Singapore, 2002.

## I.2 Analysis I

Total marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

The algebraic and order properties of  $R$ , suprema and infima, the completeness property of  $R$ , the Archimedean property, density of rational numbers in  $R$ , characterization of intervals, neighborhoods, open sets, closed sets, limit points of a set, isolated points, closure, complements, idea of uncountability of  $R$ .

*References:*

[1]: Chapter 2 (Sections 2.1-2.4, 2.5 (up to 2.5.1)), Chapter 11 (Section 11.1 (up to 11.1.6 and 11.1.8)).

[2]: Chapter 1 (Sections 1-5).

Sequences, bounded sequence, limit of a sequence, convergent sequences, limit theorems, monotone sequences, monotone convergence theorem, subsequences, convergence and divergence criteria, existence of monotonic subsequences (idea only), Bolzano-Weierstrass theorem for sequences and sets, definition of Cauchy sequence, Cauchy's convergence criterion, limit superior and limit inferior of a sequence.

*References:*

[1]: Chapter 3 (Sections 3.1-3.5, up to 3.5.6).

[2]: Chapter 2 (Sections 7-12)

Definition of infinite series, sequence of partial sums, convergence of infinite series, Cauchy criterion, absolute and conditional convergence, convergence via boundedness of sequence of partial sums, tests of convergence: comparison test, limit comparison test, ratio test, Cauchy's nth root test (proof based on limit superior), integral test (without proof), alternating series, Leibniz test.

*Reference:*

[2]: Chapter 2 (Sections 14-15)

#### **REFERENCES:**

1. **R. G. Bartle** and **D. R. Sherbert**, *Introduction to Real Analysis* (3rd Edition), John Wiley and Sons (Asia) Pte. Ltd., Singapore, 2002.
2. **K. A. Ross**, *Elementary Analysis: The Theory of Calculus*, Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2004.

### **I.3 Algebra I**

Total Marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Polar representation of complex numbers, the  $n$ th root of unity, some simple geometric notions and properties, conditions in collinearity, orthogonality and concyclicity, similar triangles, equilateral triangles, some analytic geometry in the complex plane, the circle, statement of the fundamental theorem of algebra and its consequences, Descartes' rule of signs, bound on the real zeros, interpreting the coefficients of a polynomial.

*References:*

[1]: Chapter 2, Chapter 3.

[2]: Chapter 4 (Sections 4.4, 4.6 (4.6.1-4.6.8)), Chapter 5 (Sections 5.2.7, 5.2.12), Chapter 6 (Section 6.1)

Sets, binary relations, equivalence relations, congruence relation between integers, finite product of sets, functions, composition of functions, bijective functions, invertible functions, introduction of finite and infinite sets through correspondence, binary operations, principle of mathematical induction, well-ordering property of positive integers, division algorithm, statement of fundamental theorem of arithmetic.

*References:*

[3]: Chapter 0.

[4]: Chapter 2 (Sections 2.1-2.4), Chapter 3, Chapter 4 (Section 4.4 up to Def. 4.4.6).

Systems of linear equations, row reduction and echelon forms, vector equations, the matrix equation  $Ax = b$ , solution sets of linear systems, applications of linear systems, linear independence, introduction to linear transformations, the matrix of a linear transformation, inverse of a matrix, characterizations of invertible matrices, partitioned matrices, subspaces of  $R^n$ , bases and dimension of subspaces of  $R^n$ .

*Reference:*

[5]: Chapter 1 (Sections 1.1-1.9) and Chapter 2 (Sections 2.1-2.4, 2.8-2.9).

## REFERENCES:

1. **Titu Andreescu** and **Dorin Andrica**, *Complex Numbers from A to ... Z*, Birkhauser, 2006.
2. **E.J. Barbeau**, *Polynomials*, Springer Verlag, 2003.
3. **Joseph A. Gallian**, *Contemporary Abstract Algebra* (4<sup>th</sup> Edition), Narosa Publishing House, New Delhi, 1999.
4. **Edgar G. Goodaire** and **Michael M. Parmenter**, *Discrete Mathematics with Graph Theory* (2<sup>nd</sup> Edition), Pearson Education (Singapore) Pvt. Ltd., Indian Reprint, 2003.

5. **David C. Lay**, *Linear Algebra and its Applications* (3<sup>rd</sup> Edition), Pearson Education Asia, Indian Reprint, 2007.

**I.4 Credit course I**

## **SEMESTER II**

### **II.1 Differential Equations and Mathematical Modeling I**

Total marks: 150

Theory: 75

Practical: 50

Internal Assessment: 25

5 Lectures, 2 Practicals, 1 Tutorial (per week per student)

Differential equations and mathematical models, order and degree of a differential equation, exact differential equations and integrating factors of first order differential equations, reducible second order differential equations, application of first order differential equations to acceleration-velocity model, growth and decay model.

*References:*

[2]: Chapter 1 (Sections 1.1, 1.4, 1.6), Chapter 2 (Section 2.3)

[3]: Chapter 2.

Introduction to compartmental models, lake pollution model (with case study of Lake Burley Griffin), drug assimilation into the blood (case of a single cold pill, case of a course of cold pills, case study of alcohol in the bloodstream), exponential growth of population, limited growth of population, limited growth with harvesting.

*Reference:*

[1]: Chapter 2 (Sections 2.1, 2.5-2.8), Chapter 3 (Sections 3.1-3.3)

General solution of homogeneous equation of second order, principle of superposition for a homogeneous equation, Wronskian, its properties and applications, Linear homogeneous and non-homogeneous equations of higher order with constant coefficients, Euler's equation, method of undetermined coefficients, method of variation of parameters, applications of second order differential equations to mechanical vibrations.

*Reference:*

[2]: Chapter 3 (Sections 3.1-3.5).

Equilibrium points, interpretation of the phase plane, predator-prey model and its analysis, competing species and its analysis, epidemic model of influenza and its analysis, battle model and its analysis.

*Reference:*

[1]: Chapter 5 (Sections 5.1, 5.3-5.4, 5.6-5.7), Chapter 6.

***Practical / Lab work to be performed on a computer:***

Modeling of the following problems using *Matlab / Mathematica / Maple* etc.

- (i) Plotting second and third order solution families
- (ii) Acceleration-velocity model
- (iii) Growth and decay model (both exponential and logistic)
- (iv) Any two of the following
  - (a) Lake pollution model (with constant/ seasonal flow and pollution concentration)
  - (b) Case of a single cold pill and a course of cold pills
  - (c) Case study of alcohol in the bloodstream (initial input/ continuous input on empty stomach and with substantial meal)
  - (d) Limited growth of population (with and without harvesting)
- (v) Any two of the following
  - (a) Predator prey model (basic Lotka volterra model, with density dependence, effect of DDT, two prey one predator)
  - (b) Epidemic model of influenza (basic epidemic model, contagious for life, disease with carriers, disease with re-infection, density dependent contact rate)
  - (c) Battle model (basic battle model, jungle warfare, with desertion, long range weapons)
- (vi) Taylor and Maclaurin series of  $\sin x$ ,  $\cos x$ ,  $\log(1+x)$ ,  $e^x$ ,  $(1+x)^n$ , maxima and minima, inverse of graphs.

**REFERENCES:**

1. **Belinda Barnes** and **Glenn R. Fulford**, *Mathematical Modeling with Case Studies, A Differential Equation Approach Using Maple*, Taylor and Francis, London and New York, 2002.
2. **C. H. Edwards** and **D. E. Penny**, *Differential Equations and Boundary Value Problems: Computing and Modeling*, Pearson Education, India, 2005.
3. **S. L. Ross**, *Differential Equations*, John Wiley and Sons, India, 2004.

## II.2 Analysis II

Total marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Limits of functions, sequential criterion for limits, divergence criteria, review of limit theorems and one-sided limits, continuous functions, sequential criterion for continuity, discontinuity criterion, Dirichlet's nowhere continuous function (illustrations), combinations of continuous functions and compositions of continuous functions, continuous functions on intervals, boundedness theorem, the maximum-minimum theorem, location of roots theorem, Bolzano's intermediate value theorem, intermediate value property, preservation of interval property.

*References:*

[1]: Chapter 4 (Sections 4.1-4.3), Chapter 5 (Sections 5.1-5.3).

[2]: Chapter 3 (Sections 17, 18 and 20).

Uniform continuity, uniform continuity theorem, differentiation, derivative, combinations of differentiable functions, Caratheodory theorem, chain rule, derivative of inverse functions, interior extremum theorem, intermediate value property for derivatives (Darboux's theorem), review of Rolle's theorem, mean value theorem, Cauchy's mean value theorem.

*References:*

[1]: Chapter 5 (Section 5.4 up to 5.4.3), Chapter 6 (Sections 6.1-6.2, 6.3.2).

[2]: Chapter 3 (Section 19), Chapter 5 (Sections 28, 29)

Taylor's theorem with Lagrange and Cauchy form of remainders, binomial series theorem, Taylor series, Maclaurin series, expansions of exponential, logarithmic and trigonometric functions, convex functions, applications of mean value theorems and Taylor's theorem to monotone functions. Power series, radius of convergence, interval of convergence

*References:*

[1]: Chapter 6 (Sections 6.4 (up to 6.4.6)), Chapter 9 (Section 9.4 (page 271)).

[2]: Chapter 5 (Sections 31).

### REFERENCES:

1. **R. G. Bartle** and **D. R. Sherbert**, *Introduction to Real Analysis* (3<sup>rd</sup> Edition), John Wiley and Sons (Asia) Pvt. Ltd., Singapore, 2002.
2. **K. A. Ross**, *Elementary Analysis: The Theory of Calculus*, Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2004.



## II.3 Probability and Statistics

Total marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Sample space, probability axioms, real random variables (discrete and continuous), cumulative distribution function, probability mass/density functions, mathematical expectation, moments, moment generating function, characteristic function, discrete distributions: uniform, binomial, Poisson, geometric, negative binomial, continuous distributions: uniform, normal, exponential.

*References:*

[1]: Chapter 1 (Sections 1.1, 1.3, 1.5-1.9).

[2]: Chapter 5 (Sections 5.1-5.5, 5.7), Chapter 6 (Sections 6.2-6.3, 6.5-6.6).

Joint cumulative distribution function and its properties, joint probability density functions, marginal and conditional distributions, expectation of function of two random variables, conditional expectations, independent random variables, bivariate normal distribution, correlation coefficient, joint moment generating function (jmgf) and calculation of covariance (from jmgf), linear regression for two variables.

*References:*

[1]: Chapter 2 (Sections 2.1, 2.3-2.5).

[2]: Chapter 4 (Exercise 4.47), Chapter 6 (Section 6.7), Chapter 14 (Sections 14.1, 14.2).

Chebyshev's inequality, statement and interpretation of (weak) law of large numbers and strong law of large numbers, Central Limit theorem for independent and identically distributed random variables with finite variance, Markov Chains, Chapman-Kolmogorov equations, classification of states.

*References:*

[2]: Chapter 4 (Section 4.4).

[3]: Chapter 2 (Section 2.7), Chapter 4 (Sections 4.1-4.3).

### REFERENCES:

1. **Robert V. Hogg, Joseph W. McKean and Allen T. Craig**, *Introduction to Mathematical Statistics*, Pearson Education, Asia, 2007.
2. **Irwin Miller and Marylees Miller**, *John E. Freund's Mathematical Statistics with Applications* (7th Edition), Pearson Education, Asia, 2006.
3. **Sheldon Ross**, *Introduction to Probability Models* (9th Edition), Academic Press, Indian Reprint, 2007.

### SUGGESTED READING:

1. **Alexander M. Mood, Franklin A. Graybill and Duane C. Boes**, *Introduction to the Theory of Statistics*, (3<sup>rd</sup> Edition), Tata McGraw- Hill, Reprint 2007

## **II.4 Credit course II**

## SEMESTER III

### III.1 Calculus II

Total marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Functions of several variables, level curves and surfaces, graphs of functions of two variables, limits and continuity of functions of two and three real variables, partial differentiation (two variables), partial derivative as a slope, partial derivative as a rate, higher order partial derivatives (notion only), equality of mixed partials, tangent planes, approximations and differentiability, sufficient condition for differentiability (statement only), chain rule for one and two independent parameters, illustration of chain rule for a function of three variables with three independent parameters, directional derivatives and the gradient, extrema of functions of two variables, method of Lagrange multipliers, constrained optimization problems, Lagrange multipliers with two parameters.

*Reference:*

[1]: Chapter 11.

Double integration over rectangular region, double integration over nonrectangular region, double integrals in polar co-ordinates, triple integrals, cylindrical and spherical co-ordinates, change of variables.

*Reference:*

[1]: Chapter 12.

Divergence and curl, line integrals, The Fundamental Theorem and path independence, Green's Theorem, surface integrals, Stoke's Theorem, The Divergence Theorem.

*Reference:*

[1]: Chapter 13.

#### REFERENCE:

1. **M. J. Strauss, G. L. Bradley and K. J. Smith**, *Calculus* (3<sup>rd</sup> Edition), Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2007.

#### SUGGESTED READING:

1. **Jerrold E. Marsden, Anthony J. Tromba and Alan Weinstein**, *Basic Multivariable Calculus*, Springer-Verlag, 2005.

## III.2 Numerical Methods and Programming

Total marks: 150

Theory: 75

Practical: 50

Internal Assessment: 25

5 Lectures, 2 Practicals, 1 Tutorial (per week per student)

Algorithms, Convergence, Bisection method, False position method, Fixed point iteration method, Newton's method, Secant method, LU decomposition, Gauss-Jacobi, Gauss-Siedel and SOR iterative methods.

*Reference:*

[1]: Chapter 1 (Sections 1.1-1.2), Chapter 2 (Sections 2.1-2.5), Chapter 3 (Section 3.5, 3.8).

Lagrange and Newton interpolation: linear and higher order, finite difference operators.

*References:*

[1]: Chapter 5 (Sections 5.1, 5.3)

[2]: Chapter 4 (Section 4.3).

Numerical differentiation: forward difference, backward difference and central difference.

Integration: trapezoidal rule, Simpson's rule, Euler's method.

*Reference:*

[1]: Chapter 6 (Sections 6.2, 6.4), Chapter 7 (Section 7.2)

**Note:** Emphasis is to be laid on the algorithms of the above numerical methods.

### ***Practical / Lab work to be performed on a computer:***

Use of computer aided software (CAS), for example *Matlab / Mathematica / Maple / Maxima* etc., for developing the following Numerical programs:

- (i) Calculate the sum  $1/1 + 1/2 + 1/3 + 1/4 + \dots + 1/N$ .
- (ii) To find the absolute value of an integer.
- (iii) Enter 100 integers into an array and sort them in an ascending order.
- (iv) Any two of the following
  - (a) Bisection Method
  - (b) Newton Raphson Method
  - (c) Secant Method
  - (d) Regulai Falsi Method
- (v) LU decomposition Method

- (vi) Gauss-Jacobi Method
- (vii) SOR Method or Gauss-Siedel Method
- (viii) Lagrange Interpolation or Newton Interpolation
- (ix) Simpson's rule.

**Note:** For any of the CAS *Matlab / Mathematica / Maple / Maxima* etc., Data types-simple data types, floating data types, character data types, arithmetic operators and operator precedence, variables and constant declarations, expressions, input/output, relational operators, logical operators and logical expressions, control statements and loop statements, Arrays should be introduced to the students.

**REFERENCES:**

1. **B. Bradie**, *A Friendly Introduction to Numerical Analysis*, Pearson Education, India, 2007.
2. **M. K. Jain, S. R. K. Iyengar** and **R. K. Jain**, *Numerical Methods for Scientific and Engineering Computation*, New age International Publisher, India, 5<sup>th</sup> edition, 2007.

**SUGGESTED READING:**

1. **C. F. Gerald** and **P. O. Wheatley**, *Applied Numerical Analysis*, Pearson Education, India, 7<sup>th</sup> edition, 2008

### III.3 Algebra II

Total Marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Symmetry of a square, dihedral groups, definition and examples of groups including permutation groups and quaternion groups (illustration through matrices), elementary properties of groups, subgroups and examples of subgroups, centralizer, normalizer, center of a group, cyclic groups, generators of cyclic groups, classification of subgroups of cyclic groups.

*Reference:*

[1]: Chapters 1, Chapter 2, Chapter 3 (including Exercise 20 on page 66 and Exercise 2 on page 86), Chapter 4, Chapter 5 (upto Example 3).

Cycle notation for permutations, properties of permutations, even and odd permutations, alternating group, a Check-Digit Scheme based on the dihedral group  $D_5$ , product  $(HK)$  of two subgroups, definition and properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem, an application of cosets to permutation groups, the rotation group of a cube and a soccer ball, definition and examples of the external direct product of a finite number of groups, normal subgroups, factor groups, applications of factor groups to the alternating group  $A_4$ , commutator subgroup.

*Reference:*

[1]: Chapter 5, Chapter 7 (including Exercises 3, 6 and 7 on page 168), Chapter 8 (upto Example 2), Chapter 9 (upto Example 13 and including Exercise 52 on page 188).

Definition and examples of homomorphism, properties of homomorphism, definition and examples of isomorphism, Cayley's theorem, properties of isomorphism, Isomorphism theorems I, II and III, definition and examples of automorphisms, inner automorphisms, automorphisms and inner automorphisms group, automorphism group of finite and infinite cyclic groups, applications of factor groups to automorphisms groups, Cauchy's theorem for finite abelian groups.

*Reference:*

[1]: Chapter 6, Chapter 9 (Theorems 9.3-9.5), Chapter 10

#### REFERENCES:

1. **Joseph A. Gallian**, *Contemporary Abstract Algebra* (4<sup>th</sup> Ed.), Narosa Publishing House, 1999.

#### SUGGESTED READING:

1. **David S. Dummit** and **Richard M. Foote**, *Abstract Algebra* (2<sup>nd</sup> Edition), John Wiley and Sons (Asia) Pvt. Ltd, Singapore, 2003.

### **III.4 Qualifying paper**

## SEMESTER IV

### IV.1 Differential Equations and Mathematical Modeling II

Total marks: 150

Theory: 75

Practical: 50

Internal Assessment: 25

5 Lectures, 2 Practicals, 1 Tutorial (per week per student)

Introduction, classification, construction and geometrical interpretation of first order partial differential equations (PDE), method of characteristic and general solution of first order PDE, canonical form of first order PDE, method of separation of variables for first order PDE.

*Reference:*

[1]: Chapter 2.

Mathematical modeling of vibrating string, vibrating membrane, conduction of heat in solids, gravitational potential, conservation laws and Burger's equations, classification of second order PDE, reduction to canonical forms, equations with constant coefficients, general solution.

*Reference:*

[1]: Chapter 3 (Sections 3.1-3.3, 3.5-3.7), Chapter 4.

Cauchy problem for second order PDE, homogeneous wave equation, initial boundary value problems, non-homogeneous boundary conditions, finite strings with fixed ends, non-homogeneous wave equation, Riemann problem, Goursat problem, spherical and cylindrical wave equation.

*Reference:*

[1]: Chapter 5.

Method of separation of variables for second order PDE, vibrating string problem, existence and uniqueness of solution of vibrating string problem, heat conduction problem, existence and uniqueness of solution of heat conduction problem, Laplace and beam equation, non-homogeneous problem.

*Reference:*

[1]: Chapter 7.



**Practical / Lab work to be performed on a computer:**

Modeling of the following problems using *Matlab / Mathematica / Maple* etc.

(i) Solution of Cauchy problem for first order PDE.

(ii) Finding the characteristics for the first order PDE.

(iii) Plot the integral surfaces of a given first order PDE with initial data.

(iv) Solution of wave equation  $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$  for any two of the following associated conditions

(a)  $u(x,0) = \varphi(x), \quad u_t(x,0) = \psi(x), \quad x \in R \quad t > 0$

(b)  $u(x,0) = \varphi(x), \quad u_t(x,0) = \psi(x), \quad u(0,t) = 0, \quad x \in (0,\infty), \quad t > 0$

(c)  $u(x,0) = \varphi(x), \quad u_t(x,0) = \psi(x), \quad u_x(0,t) = 0, \quad x \in (0,\infty), \quad t > 0$

(d)  $u(x,0) = \varphi(x), \quad u_t(x,0) = \psi(x), \quad u(0,t) = 0, \quad u(l,t) = 0, \quad 0 < x < l, \quad t > 0$

(v) Solution of the heat equation  $\frac{\partial u}{\partial t} - \kappa \frac{\partial^2 u}{\partial x^2} = 0$  for any two of the following associated conditions

(a)  $u(x,0) = \varphi(x), \quad u(0,t) = a, \quad u(l,t) = b, \quad 0 < x < l, \quad t > 0$

(b)  $u(x,0) = \varphi(x), \quad x \in R, \quad 0 < t < T$

(c)  $u(x,0) = \varphi(x), \quad u(0,t) = a, \quad x \in (0,\infty), \quad t \geq 0$

**REFERENCE:**

1. **Tyn Myint-U and Lokenath Debnath**, *Linear Partial Differential Equation for Scientists and Engineers*, Springer, Indian reprint, 2006.

**SUGGESTED READING:**

1. **Ioannis P Stavroulakis and Stepan A Tersian**, *Partial Differential Equations: An Introduction with Mathematica and MAPLE*, World Scientific, Second Edition, 2004.

## IV.2 Analysis III

Total marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Riemann integral, basic inequality of Riemann integral, Riemann condition of integrability, Riemann sum, algebraic and order properties of the Riemann integral, Riemann integrability for continuous functions, monotonic functions and functions with finite number of discontinuities (without proof), the fundamental theorem of calculus (second fundamental theorem without proof), consequences of the fundamental theorem of calculus: integration by parts and change of variables, mean value theorem of calculus (statement only). improper integrals, convergence of improper integrals, tests of convergence for improper integrals, Abel's and Dirichlet's tests for improper integrals, Beta and Gamma functions and their relations.

*References:*

[3]: Chapter 6 (Articles 32-34, 36).

[2]: Chapter 9 (Sections 9.4-9.6)

Pointwise and uniform convergence of sequence of functions, uniform convergence and continuity, uniform convergence and differentiation, uniform convergence and integration, Cauchy criterion for uniform convergence, series of functions and convergence, Weierstrass M-test, Weierstrass approximation theorem (statement only). Differentiation and integration of Power series, Abel's theorem (without proof), exponential and logarithmic functions

*References:*

[1]: Chapter 8 (Sections 8.1, 8.2.3, 8.2.4), Chapter 9 (Sections 9.4.1-9.4.6).

[3]: Chapter 4 (Sections 24-27), Chapter 6 (Section 37)

### REFERENCES:

1. **R. G. Bartle** and **D. R. Sherbert**, *Introduction to Real Analysis* (3rd Edition), John Wiley and Sons (Asia) Pvt. Ltd., Singapore, 2002.
2. **Sudhir R. Ghorpade** and **Balmohan V. Limaye**, *A Course in Calculus and Real Analysis*, Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2006.
3. **K. A. Ross**, *Elementary Analysis: The Theory of Calculus*, Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2004.

### IV.3 Algebra III

Total Marks: 100

Theory: 75

Internal Assessment: 25

5 Lecture, 1 Tutorial (per week per student)

Definition and examples of rings, properties of rings, subrings, integral domains, definition and examples of fields, characteristic of a ring, ideals, ideal generated by subsets in a commutative ring with unity, factor rings, operations on ideals, prime ideals and maximal ideals, definition and examples of ring homomorphisms, properties of ring homomorphisms, isomorphisms, isomorphism theorems I, II and III, field of quotients.

*Reference:*

[2]: Chapter 12, Chapter 13, Chapter 14, Chapter 15.

Definition of polynomial rings over commutative rings, the division algorithm and consequences, principal ideal domains, factorization of polynomials, reducibility tests, irreducibility tests, Eisenstein criterion, unique factorization in  $Z[x]$ , an application of unique factorization to weird dice, divisibility in integral domains, irreducibles, primes, unique factorization domains, Euclidean domains.

*Reference:*

[2]: Chapter 16, Chapter 17, Chapter 18.

Vector spaces, subspaces, algebra of subspaces, quotient spaces, linear combinations and systems of linear equations, linear span, linear independence, basis and dimension, dimensions of subspaces, linear transformations, null space, range, rank and nullity of linear transformations, matrix of a linear transformation, algebra of linear transformations, isomorphism, Isomorphism theorems, invertibility and isomorphisms, change of basis.

*Reference:*

[1]: Chapter 1 (Sections 1.2-1.6), Chapter 2 (Sections 2.1-2.5).

#### REFERENCES:

1. **Stephen H. Friedberg, Arnold J. Insel and Lawrence E. Spence**, *Linear Algebra* (4<sup>th</sup> Edition), Prentice-Hall of India Pvt. Ltd., New Delhi, 2004.
2. **Joseph A. Gallian**, *Contemporary Abstract Algebra* (4<sup>th</sup> Edition), Narosa Publishing House, New Delhi, 1999.

## **IV.4 Credit course III.1**

## SEMESTER V

### V.1 Differential Equations and Mathematical Modeling III

Total marks: 150

Theory: 75

Practical: 50

Internal Assessment: 25

5 Lectures, 2 Practicals, 1 Tutorial (per week per student)

Power series solution of a differential equation about an ordinary point, solution about a regular singular point, Bessel's equation and Legendre's equation, Laplace transform and inverse transform, application to initial value problem up to second order.

*Reference:*

[2]: Chapter 7 (Sections 7.1-7.3), Chapter 8 (Sections 8.2-8.3).

Monte Carlo Simulation Modeling: simulating deterministic behavior (area under a curve, volume under a surface), Generating Random Numbers: middle square method, linear congruence, Queuing Models: harbor system, morning rush hour, Overview of optimization modeling, Linear Programming Model: geometric solution algebraic solution, simplex method, sensitivity analysis

*Reference:*

[3]: Chapter 5 (Sections 5.1-5.2, 5.5), Chapter 7.

Graphs, diagraphs, networks and subgraphs, vertex degree, paths and cycles, regular and bipartite graphs, four cube problem, social networks, exploring and traveling, Eulerian and Hamiltonian graphs, applications to dominoes, diagram tracing puzzles, Knight's tour problem, gray codes.

*Reference:*

[1]: Chapter 1 (Section 1.1), Chapter 2, Chapter 3.

**Note:** Chapter 1 (Section 1.1), Chapter 2 (Sections 2.1-2.4), Chapter 3 (Sections 3.1-3.3) are to be reviewed only. This is in order to understand the models on Graph Theory.

***Practical / Lab work to be performed on a computer:***

Modeling of the following problems using *Matlab / Mathematica / Maple* etc.

(i) Plotting of Legendre polynomial for  $n = 1$  to 5 in the interval  $[0,1]$ . Verifying graphically that all the roots of  $P_n(x)$  lie in the interval  $[0,1]$ .

- (ii) Automatic computation of coefficients in the series solution near ordinary points
- (iii) Plotting of the Bessel's function of first kind of order 0 to 3.
- (iv) Automating the Frobenius Series Method
- (v) Random number generation and then use it for one of the following
  - (a) Simulate area under a curve
  - (b) Simulate volume under a surface
- (vi) Programming of either one of the queuing model
  - (a) Single server queue (e.g. Harbor system)
  - (b) Multiple server queue (e.g. Rush hour)
- (vii) Programming of the Simplex method for 2/3 variables

**REFERENCES:**

1. **Joan M. Aldous** and **Robin J. Wilson**, *Graphs and Applications: An Introductory Approach*, Springer, Indian reprint, 2007.
2. **Tyn Myint-U** and **Lokenath Debnath**, *Linear Partial Differential Equation for Scientists and Engineers*, Springer, Indian reprint, 2006.
3. **Frank R. Giordano**, **Maurice D. Weir** and **William P. Fox**, *A First Course in Mathematical Modeling*, Thomson Learning, London and New York, 2003.

## V.2 Analysis IV

Total marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Definition and examples of metric spaces, isometries, diameter, isolated points, accumulation and boundary points, closure and interior, open and closed sets, Cantor's intersection theorem, open and closed balls, convergence, Cauchy sequence and boundedness.

*References:*

[1]: Chapter 1 (Sections 1.1 (up to example 1.1.17), 1.3-1.4, 1.6-1.7), Chapter 2 (Sections 2.1-2.3, 2.5-2.6), Chapter 3 (Sections 3.1, 3.6-3.7), Chapter 4 (Sections 4.1-4.4, 4.7), Chapter 5 (Sections 5.1-5.3), Chapter 6 (Sections 6.1-6.2, 6.4-6.8), Chapter 7 (Sections 7.1, 7.4, 7.6-7.8).

Continuity and uniform continuity, completeness, contraction mapping theorem, Baire's category theorem.

*Reference:*

[1]: Chapter 8 (Sections 8.1-8.3, 8.5, 8.9-8.10), Chapter 9 (Sections 9.1 (up to Subsection 9.1.3), 9.2 (Theorem 9.2.1 with 1st two criteria), 9.4, 9.9), Chapter 10 (Sections 10.2 (only Cauchy criterion), 10.3, 10.8, 10.10).

Connectedness, connected subsets, connected components, pathwise connectedness.

*Reference:*

[1]: Chapter 11 (Sections 11.1 – 11. 8)

Compactness, compact subsets, compactness of products.

*Reference:*

[1]: Chapter 12 (Sections 12.1 – 12. 5).

### REFERENCES:

1. **Mícheál Ó Searcóid**, *Metric Spaces*, Springer Undergraduate Mathematics Series, Springer-Verlag, London Limited, London, 2007.
2. **G. F. Simmons**, *Introduction to Topology and Modern Analysis*, Mcgraw-Hill Editions, 2004.

## V.3 Algebra IV

Total Marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Dual spaces, dual basis, double dual, transpose and its matrix in the dual basis, annihilators, eigenvalues and eigenvectors, characteristic polynomial, diagonalizability, invariant subspaces and the Cayley-Hamilton theorem, the minimal polynomial for a linear transformation.

*Reference:*

[1]: Chapter 2 (Section 2.6), Chapter 5 (Sections 5.1-5.2, 5.4, 7.3).

Inner product spaces and norms, Gram-Schmidt orthogonalisation process, orthogonal complements, Bessel's inequality, adjoints of linear operators, least square approximations, minimal solutions to system of linear equations.

*Reference:*

[1]: Chapter 6 (Sections 6.1-6.3).

Extension fields, fundamental theorem of field theory, splitting fields, zeros of an irreducible polynomial, perfect fields, characterization of extensions, algebraic extensions, finite extensions, properties of algebraic extensions, classification of finite fields, structure of finite fields, subfields of finite fields, constructible numbers, straight edge and compass construction.

*Reference:*

[2]: Chapter 20, Chapter 21, Chapter 22, Chapter 23.

### REFERENCES:

1. **Stephen H. Friedberg, Arnold J. Insel and Lawrence E. Spence**, *Linear Algebra* (4<sup>th</sup> Edition), Prentice-Hall of India Pvt. Ltd., New Delhi, 2004.
2. **Joseph A. Gallian**, *Contemporary Abstract Algebra* (4<sup>th</sup> Edition), Narosa Publishing House, New Delhi, 1999.



## V.4 Linear Programming and Theory of Games

Total Marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Introduction to linear programming problem, Theory of simplex method, optimality and unboundedness, the simplex algorithm, simplex method in tableau format, introduction to artificial variables, two-phase method, Big-M method and their comparison.

*Reference:*

[1]: Chapter 3 (Sections 3.2-3.3, 3.5-3.8), Chapter 4 (Sections 4.1-4.4).

Duality, formulation of the dual problem, primal-dual relationships, economic interpretation of the dual.

*Reference:*

[1]: Chapter 6 (Sections 6.1- 6.3).

Transportation problem and its mathematical formulation, northwest-corner method least cost method and Vogel approximation method for determination of starting basic solution, algorithm for solving transportation problem, assignment problem and its mathematical formulation, Hungarian method for solving assignment problem.

*Reference:*

[3]: Chapter 5 (Sections 5.1, 5.3-5.4).

Game theory: formulation of two person zero sum games, solving two person zero sum games, games with mixed strategies, graphical solution procedure, linear programming solution of games.

*Reference:*

[2]: Chapter 14.

### REFERENCES:

1. **Mokhtar S. Bazaraa, John J. Jarvis and Hanif D. Sherali**, *Linear Programming and Network Flows* (2nd edition), John Wiley and Sons, India, 2004.
2. **F. S. Hillier and G. J. Lieberman**, *Introduction to Operations Research* (9th Edition), Tata McGraw Hill, Singapore, 2009.
3. **Hamdy A. Taha**, *Operations Research, An Introduction* (8th edition), Prentice-Hall India, 2006.

### SUGGESTED READING:

1. **G. Hadley**, *Linear Programming*, Narosa Publishing House, New Delhi, 2002.

## SEMESTER VI

### VI.1 Analysis V

Total marks: 150

Theory: 75

Internal Assessment: 25

Practical: 50

5 Lectures, Practical 2,1 Tutorial (per week per student)

Review of complex plane, sequences and series, connected sets and polygonally connected sets in the complex plane, stereographic projection, analytic polynomials, power series, analytic functions, Cauchy-Riemann equations, functions  $e^z$ ,  $\sin z$  and  $\cos z$ .

*Reference:*

[1]: Chapter 1, Chapter 2, Chapter 3.

Line integrals and their properties, closed curve theorem for entire functions, Cauchy integral formula and Taylor expansions for entire functions, Liouville's theorem and the fundamental theorem of algebra.

*Reference:*

[1]: Chapter 4, Chapter 5.

Power series representation for functions analytic in a disc, analyticity in an arbitrary open set, uniqueness theorem, definitions and examples of conformal mappings, bilinear transformations.

*Reference:*

[1]: Chapter 6 (Sections 6.1-6.2, 6.3 (up to theorem 6.9), Chapter 9 (Sections 9.2, 9.7-9.8, 9.9 (statement only), 9.10, 9.11 (with examples), 9.13), Chapter 13 (Sections 13.1, 13.2 (up to Theorem 13.11 including examples)).

Fourier series, Piecewise continuous functions, Fourier cosine and sine series, property of Fourier coefficients, Fourier theorem, discussion of the theorem and its corollary.

*Reference:* [2].

#### ***Practical / Lab work to be performed on a computer:***

Modeling of the following problems using *Matlab / Mathematica / Maple* etc.

(i) Declaring a complex number e.g.  $z_1 = 3 + 4i$ ,  $z_2 = 4 - 7i$ . Discussing their algebra  $z_1 + z_2$ ,  $z_1 - z_2$ ,  $z_1 * z_2$ , and  $z_1 / z_2$  and then plotting them.

(ii) Finding conjugate, modulus and phase angle of an array of complex numbers. e.g.,  $Z = [2+ 3i \ 4-2i \ 6+11i \ 2-5i]$

- (iii) Compute the integral over a straight line path between the two specified end points e.g.,  $\int_C f(z) dz$ , where  $C$  is the straight line path from  $a + ib$  to  $c + id$ .
- (iv) Perform contour integration e.g.  $\int_C f(z) dz$  where  $C$  is the Contour given by  $g(x, y) = 0$ .
- (v) Plotting of the complex functions like  $f(z) = z$ ,  $f(z) = z^3$ ,  $f(z) = (z^4 - 1)^{1/4}$ , etc.
- (vi) Finding the residues of the complex function.
- (vii) Taylor series expansion of a given function  $f(z)$  around a given point  $z$ , given the number of terms in the Taylor series expansion. Hence comparing the function and its Taylor series expansion by plotting the magnitude of each. For example
- (i)  $f(z) = \exp(z)$  around  $z = 0$ ,  $n = 40$ .
- (ii)  $f(z) = \exp(z^2)$  around  $z = 0$ ,  $n = 160$ , etc.
- (viii) To perform Laurentz series expansion of a given function  $f(z)$  around a given point  $z$ , e.g.,  $f(z) = (\sin z - 1)/z^4$  around  $z = 0$ ,  $f(z) = \cot(z)/z^4$  around  $z = 0$ , etc.
- (ix) Computing the Fourier series, Fourier sine series and Fourier cosine series of a function and plotting their graphs.

#### REFERENCES:

1. **Joseph Bak** and **Donald J. Newman**, *Complex analysis* (2<sup>nd</sup> Edition), Undergraduate Texts in Mathematics, Springer-Verlag New York, Inc., New York, 1997.
2. **John P. D'Angelo**, *An Introduction to Complex Analysis and Geometry*, American Mathematical Society, 2010
3. *Fourier Series*, Lecture notes published by the Institute of Life Long Learning, University of Delhi, Delhi, 2011.

## VI.2 Algebra V

Total Marks : 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Properties of external direct products, the group of units modulo  $n$  as an external direct product, applications of external direct products to data security, public key cryptography, definition and examples of internal direct products, fundamental theorem of finite abelian groups, definition and examples of group actions, stabilizers and kernels of group actions, permutation representation associated with a given group action.

*References:*

[1]: Chapter 1 (Section 1.7), Chapter 2 (Section 2.2), Chapter 4 (Section 4.1).

[3]: Chapter 8, Chapter 9 (Section on internal direct products), Chapter 11.

Applications of group actions: Cauchy's theorem, Index theorem, Cayley's theorem, conjugacy relation, class equation and consequences, conjugacy in  $S_n$ ,  $p$ -groups, Sylow's theorems and consequences. Definition and examples of simple groups, non-simplicity tests, composition series, Jordan-Holder theorem, solvable groups.

*References:*

[1]: Chapter 3 (Section 3.4, Exercise 9), Chapter 4 (Sections 4.2-4.3, 4.5-4.6).

[3]: Chapter 25.

Normal operators and self-adjoint operators, unitary and orthogonal operators, matrices of orthogonal and unitary operators, rigid motions, orthogonal operators on  $R^2$ , conic sections.

*Reference:*

[2]: Chapter 6 (Sections 6.4-6.5).

Primary decomposition theorem, theorem on decomposition into sum of diagonalizable and nilpotent operator, cyclic subspaces and annihilators, cyclic decomposition theorem, rational form, invariant factors, Jordan form.

*Reference:*

[4]: Chapter 6 (Section 6.8), Chapter 7 (Sections 7.1-7.3).

### REFERENCES:

1. **David S. Dummit** and **Richard M. Foote**, *Abstract Algebra* (2<sup>nd</sup> Edition), John Wiley and Sons (Asia) Pte. Ltd, Singapore, 2003.
2. **Stephen H. Friedberg**, **Arnold J. Insel** and **Lawrence E. Spence**, *Linear Algebra* (4<sup>th</sup> Edition), Prentice-Hall of India Pvt. Ltd., New Delhi, 2004.
3. **Joseph A. Gallian**, *Contemporary Abstract Algebra* (4<sup>th</sup> Edition), Narosa Publishing House, New Delhi, 1999.
4. **Kenneth Hoffman** and **Ray Kunze**, *Linear Algebra* (2<sup>nd</sup> edition), Pearson Education Inc., India, 2005.

## OPTIONAL PAPERS VI.3

### Optional Paper 1: Discrete Mathematics

Total Marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Definition, examples and basic properties of ordered sets, maps between ordered sets, duality principle, lattices as ordered sets, lattices as algebraic structures, sublattices, products and homomorphisms.

*References:*

[1]: Chapter 1 (till the end of 1.18), Chapter 2 (Sections 2.1-2.13), Chapter 5 (Sections 5.1-5.11).

[3]: Chapter 1 (Section 1).

Definition, examples and properties of modular and distributive lattices, Boolean algebras, Boolean polynomials, minimal forms of Boolean polynomials, Quinn-McCluskey method, Karnaugh diagrams, switching circuits and applications of switching circuits.

*References:*

[1]: Chapter 6.

[3]: Chapter 1 (Sections 3-4, 6), Chapter 2 (Sections 7-8).

Definition, examples and basic properties of graphs, pseudographs, complete graphs, bi-partite graphs, isomorphism of graphs, paths and circuits, Eulerian circuits, Hamiltonian cycles, the adjacency matrix, weighted graph, travelling salesman's problem, shortest path, Dijkstra's algorithm, Floyd-Warshall algorithm.

*Reference:*

[2]: Chapter 9, Chapter 10.

#### REFERENCES:

1. **B. A. Davey** and **H. A. Priestley**, *Introduction to Lattices and Order*, Cambridge University Press, Cambridge, 1990.
2. **Edgar G. Goodaire** and **Michael M. Parmenter**, *Discrete Mathematics with Graph Theory* (2nd Edition), Pearson Education (Singapore) Pte. Ltd., Indian Reprint 2003.
3. **Rudolf Lidl** and **Günter Pilz**, *Applied Abstract Algebra* (2nd Edition), Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2004.

## Optional Paper 2 : Mathematical Finance

Total Marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Basic principles: Comparison, arbitrage and risk aversion, Interest (simple and compound, discrete and continuous), time value of money, inflation, net present value, internal rate of return (calculation by bisection and Newton-Raphson methods), comparison of NPV and IRR. Bonds, bond prices and yields, Macaulay and modified duration, term structure of interest rates: spot and forward rates, explanations of term structure, running present value, floating-rate bonds, immunization, convexity, puttable and callable bonds.

*References:*

[1]: Chapter 1, Chapter 2, Chapter 3, Chapter 4.

Asset return, short selling, portfolio return, (brief introduction to expectation, variance, covariance and correlation), random returns, portfolio mean return and variance, diversification, portfolio diagram, feasible set, Markowitz model (review of Lagrange multipliers for 1 and 2 constraints), Two fund theorem, risk free assets, One fund theorem, capital market line, Sharpe index. Capital Asset Pricing Model (CAPM), betas of stocks and portfolios, security market line, use of CAPM in investment analysis and as a pricing formula, Jensen's index.

*References:*

[1]: Chapter 6, Chapter 7, Chapter 8 (Sections 8.5--8.8).

[3]: Chapter 1 (for a quick review/description of expectation etc.)

Forwards and futures, marking to market, value of a forward/futures contract, replicating portfolios, futures on assets with known income or dividend yield, currency futures, hedging (short, long, cross, rolling), optimal hedge ratio, hedging with stock index futures, interest rate futures, swaps. Lognormal distribution, Lognormal model / Geometric Brownian Motion for stock prices, Binomial Tree model for stock prices, parameter estimation, comparison of the models. Options, Types of options: put / call, European / American, pay off of an option, factors affecting option prices, put call parity.

*References:*

[1]: Chapter 10 (except 10.11, 10.12), Chapter 11 (except 11.2 and 11.8)

[2]: Chapter 3, Chapter 5, Chapter 6, Chapter 7 (except 7.10 and 7.11), Chapter 8.  
Chapter 9

[3]: Chapter 3

**REFERENCES:**

1. **David G. Luenberger**, *Investment Science*, Oxford University Press, Delhi, 1998.
2. **John C. Hull**, *Options, Futures and Other Derivatives* (6th Edition), Prentice-Hall India, Indian reprint, 2006.
3. **Sheldon Ross**, *An Elementary Introduction to Mathematical Finance* (2nd Edition), Cambridge University Press, USA, 2003.



## Optional Paper 3 : Mechanics

Total Marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Moment of a force about a point and an axis, couple and couple moment, Moment of a couple about a line, resultant of a force system, distributed force system, free body diagram, free body involving interior sections, general equations of equilibrium, two point equivalent loading, problems arising from structures, static indeterminacy.

*Reference:*

[1]: Chapter 3, Chapter 4, Chapter 5.

Laws of Coulomb friction, application to simple and complex surface contact friction problems, transmission of power through belts, screw jack, wedge, first moment of an area and the centroid, other centers, Theorem of Pappus-Guldinus, second moments and the product of area of a plane area, transfer theorems, relation between second moments and products of area, polar moment of area, principal axes.

*Reference:*

[1]: Chapter 6 (Sections 6.1-6.7), Chapter 7

Conservative force field, conservation for mechanical energy, work energy equation, kinetic energy and work kinetic energy expression based on center of mass, moment of momentum equation for a single particle and a system of particles, translation and rotation of rigid bodies, Chasles' theorem, general relationship between time derivatives of a vector for different references, relationship between velocities of a particle for different references, acceleration of particle for different references.

*Reference:*

[1]: Chapter 11, Chapter 12 (Sections 12.5-12.6), Chapter 13.

### REFERENCES:

1. **I.H. Shames** and **G. Krishna Mohan Rao**, *Engineering Mechanics: Statics and Dynamics* (4<sup>th</sup> Edition), Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2009.
2. **R.C. Hibbeler** and **Ashok Gupta**, *Engineering Mechanics: Statics and Dynamics* (11<sup>th</sup> Edition), Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi.

## Optional Paper 4 : Number Theory

Total Marks: 100

Theory: 75

Internal Assessment: 25

5 Lectures, 1 Tutorial (per week per student)

Linear Diophantine equation, prime counting function, statement of prime number theorem, Goldbach conjecture, linear congruences, complete set of residues, Chinese remainder theorem, Fermat's little theorem, Wilson's theorem.

*References:*

[1]: Chapter 2 (Section 2.5), Chapters 3 (Section 3.3), Chapter 4 (Sections 4.2 and 4.4), Chapter 5 (Section 5.2 excluding pseudoprimes, Section 5.3).

[2]: Chapter 3 (Section 3.2).

Number theoretic functions, sum and number of divisors, totally multiplicative functions, definition and properties of the Dirichlet product, the Möbius inversion formula, the greatest integer function, Euler's phi-function, Euler's theorem, reduced set of residues, some properties of Euler's phi-function.

*References:*

[1]: Chapter 6 (Sections 6.1-6.3), Chapter 7.

[2]: Chapter 5 (Section 5.2 (Definition 5.5-Theorem 5.40), Section 5.3 (Theorem 5.15-Theorem 5.17, Theorem 5.19)).

Order of an integer modulo  $n$ , primitive roots for primes, composite numbers having primitive roots, Euler's criterion, the Legendre symbol and its properties, quadratic reciprocity, quadratic congruences with composite moduli. Public key encryption, RSA encryption and decryption, the equation  $x^2 + y^2 = z^2$ , Fermat's Last Theorem.

*Reference:*

[1]: Chapters 8 (Sections 8.1-8.3), Chapter 9, Chapter 10 (Section 10.1), Chapter 12.

### REFERENCES:

1. **David M. Burton**, *Elementary Number Theory* (6th Edition), Tata McGraw-Hill Edition, Indian reprint, 2007.
2. **Neville Robinns**, *Beginning Number Theory* (2nd Edition), Narosa Publishing House Pvt. Limited, Delhi, 2007.

## **VI.4 Credit course III.2**